2.5: Bernoulli Equations (#15-22)

Form: $\frac{dy}{dx} + P(x)y = f(x)y^n$

$n=0$ $\frac{dy}{dx} + P(x)y = f(x)$ (linear)

$n=1$ Manipulation

If $n \neq 0$ & $n \neq 1$, then we substitute

$$\boxed{u = y^{1-n}}$$

(ex) #16 $\frac{dy}{dx} - y = e^x y^2$ ← Bernoulli

$n=2$ $u = y^{1-2} = y^{-1} = \frac{1}{y}$

$\frac{du}{dx} = -y^{-2}\frac{dy}{dx}$

$\frac{du}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$

$-y^2\frac{du}{dx} = \frac{dy}{dx}$

$\frac{dy}{dx} - y = e^x y^2$

$-y^2\frac{du}{dx} - y = e^x y^2$

$\frac{-y^2}{-y^2}\frac{du}{dx} + \frac{-y}{-y^2} = \frac{e^x y^2}{-y^2}$

$\frac{du}{dx} + \frac{1}{y} = -e^x$

$\frac{du}{dx} + u = -e^x$

$P(x) = 1$ $\mu(x) = e^{\int P(x)dx} = e^{\int 1\,dx} = e^x$

$$e^x \frac{du}{dx} + e^x u = -e^x e^x$$

$$\frac{d}{dx}\left[e^x u\right] = -e^{2x}$$

$$\int \frac{d}{dx} e^x u \, dx = \int -e^{2x} \, dx$$

$$e^x u = \int -e^{2x} \, dx$$

$$= \frac{-e^{2x}}{2} + C$$

$$u = \frac{-e^{2x}}{2e^x} + \frac{C}{e^x}$$

$$= \frac{-e^x}{2} + \frac{C}{e^x}$$

$$\frac{1}{y} = \frac{-e^x}{2} + \frac{C}{e^x}$$

$$y = \frac{1}{\frac{-e^x}{2} + \frac{C}{e^x}}$$