

2.5: Bernoulli Equations (#15-22)

Form: $\frac{dy}{dx} + P(x)y = f(x)y^n$

$n=0$ $\frac{dy}{dx} + P(x)y = f(x)$ (Linear)

$n=1$ Manipulation

if $n \neq 0$ & $n \neq 1$, then we substitute

$$u = y^{1-n}$$

ex) #6 $\frac{dy}{dx} - y = e^x y^2$ Bernoulli

$n=2$ $u = y^{1-2} = y^{-1} = \frac{1}{y}$

$$\frac{du}{dx} = -y^{-2} \frac{dy}{dx}$$

$$\frac{dy}{dx} - y = e^x y^2$$

$$\frac{du}{dx} = -\frac{1}{y^2} \frac{dy}{dx}$$

$$-y^2 \frac{du}{dx} - y = e^x y^2$$

$$-y^2 \frac{du}{dx} = \frac{dy}{dx}$$

$$\frac{-y^2}{-y^2} \frac{du}{dx} = \frac{dy}{dx}$$

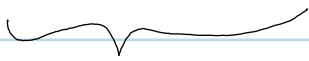
$$\frac{du}{dx} + \frac{1}{y} = -e^x$$

$$\frac{du}{dx} + u = -e^x$$

$$P(x) = 1$$

$$\mu(x) = e^{\int P(x) dx} = e^{\int 1 dx} = e^x$$

$$e^x \frac{du}{dx} + e^x u = -e^x e^x$$



$$\frac{d}{dx} [e^x u] = -e^{2x}$$

$$\int \frac{d}{dx} [e^x u] dx = \int -e^{2x} dx$$

$$e^x u = \int -e^{2x} dx$$
$$= -\frac{e^{2x}}{2} + C$$

$$u = -\frac{e^{2x}}{2e^x} + \frac{C}{e^x}$$

$$= -\frac{e^x}{2} + \frac{C}{e^x}$$

$$\frac{1}{y} = -\frac{e^x}{2} + \frac{C}{e^x}$$

$$y = \frac{1}{-\frac{e^x}{2} + \frac{C}{e^x}}$$